Massachusetts Institute of Technology
Department of Electrical Engineering and Computer Science

6.200 – Circuits & Electronics
Spring 2024

Quiz #2

24 April 2024

Name: ____________________

MIT Kerberos ID: ____________________

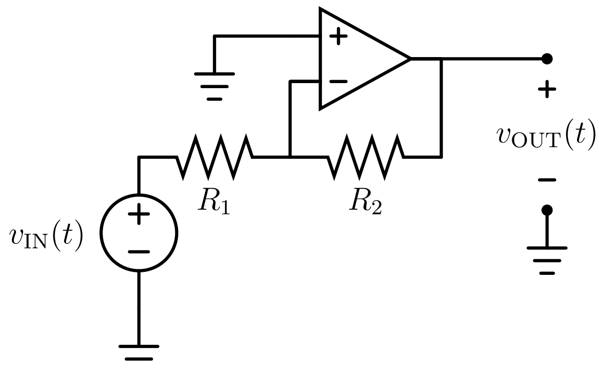
Recitation Time: 11 12 1

- There are 17 pages in this quiz, including this cover page.
- Please put your name and Kerberos ID in the spaces provided above, and circle the time of your recitation.
- Please do not remove any pages from this quiz.
- Do your work for each question within the boundaries of that question, or on the back of the preceding page. *When finished with each part, clearly write your answer for that part into the corresponding answer box or graph.*
- Make sure all work is on pages with QR codes, and **do not write on the QR codes.**
- *All numerical answers require proper units.*
- *In order to guarantee receipt of full credit, all answers should be justified by supporting math and/or explanations.*
- This is a closed-book and closed-electronics quiz, but a single two-sided page of notes is allowed.
- Good luck!

Problem 1: Op Amps & More – 20%

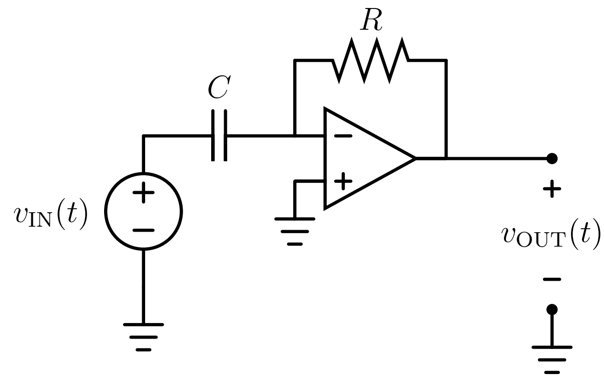
This problem focuses on three separate circuits, each containing an op amp. Assume that all op amps in this problem are ideal.

(1A) Given the circuit shown below, determine $v_{\text{OUT}}(t)$ in terms of $v_{\text{IN}}(t)$, R_1, and R_2.



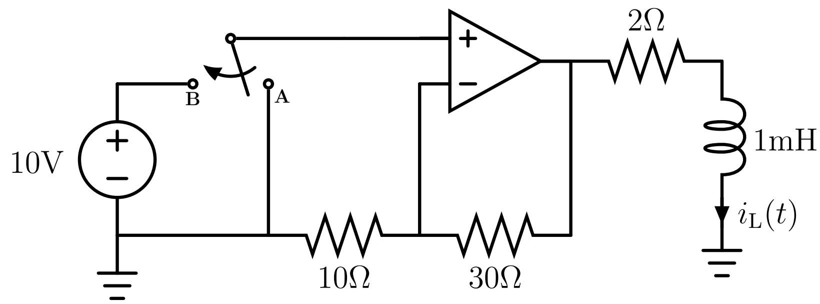
$v_{\text{OUT}}(t) =$

(1B) Given the circuit shown below, determine $v_{\mathrm{OUT}}(t)$ in terms of $v_{\mathrm{IN}}(t)$, R, and C.



$v_{\mathrm{OUT}}(t) =$

(1C) Consider the circuit shown below, in which the switch has been in Position A for a very long time. Determine the inductor current $i_L(t)$ while the switch is in Position A. *Proper units are expected.*



$i_L(t) =$

(1D) Consider again the circuit from the previous part. At $t = 0$, the switch moves from Position A to Position B. Determine the inductor current $i_{\mathrm{L}}(t)$ for $t \geq 0^+$ after the switch changes position. *Proper units are expected.*

$i_{\mathrm{L}}(t) =$

In addition, sketch $i_{\mathrm{L}}(t)$ on the axes given below. In your sketch, label all key values/asymptotes (with units); and for any portions of the graph that trace out an exponential curve, indicate the associated time constant(s).

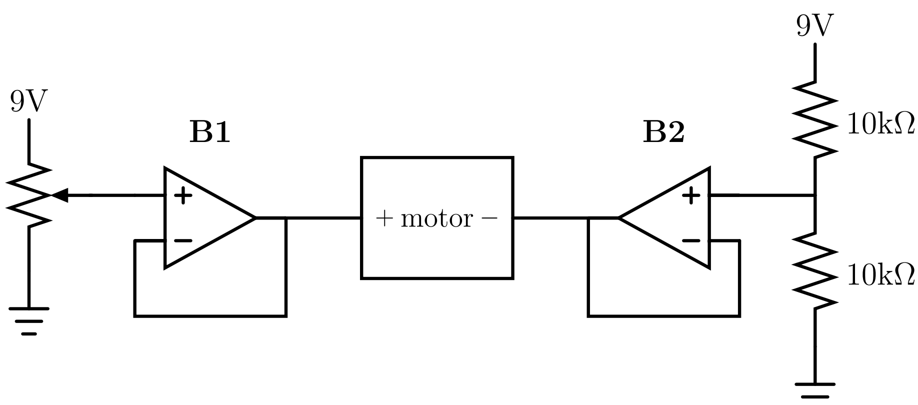
Problem 2: Motor Control – 20%

In the labs from weeks 5 and 6, we experimented with circuits that controlled motors. This problem considers three such circuits, each of which is designed to apply a voltage across a motor in proportion to a control value. The control value is created using a potentiometer as has been done in lab many times. Throughout, assume that all op amps in the circuits are ideal with the exception that they are all powered from the same 9-V source that provides power to the potentiometer. Notice that **each circuit has only one potentiometer**; all other resistors are constant-valued.

For each circuit, provide the following information.

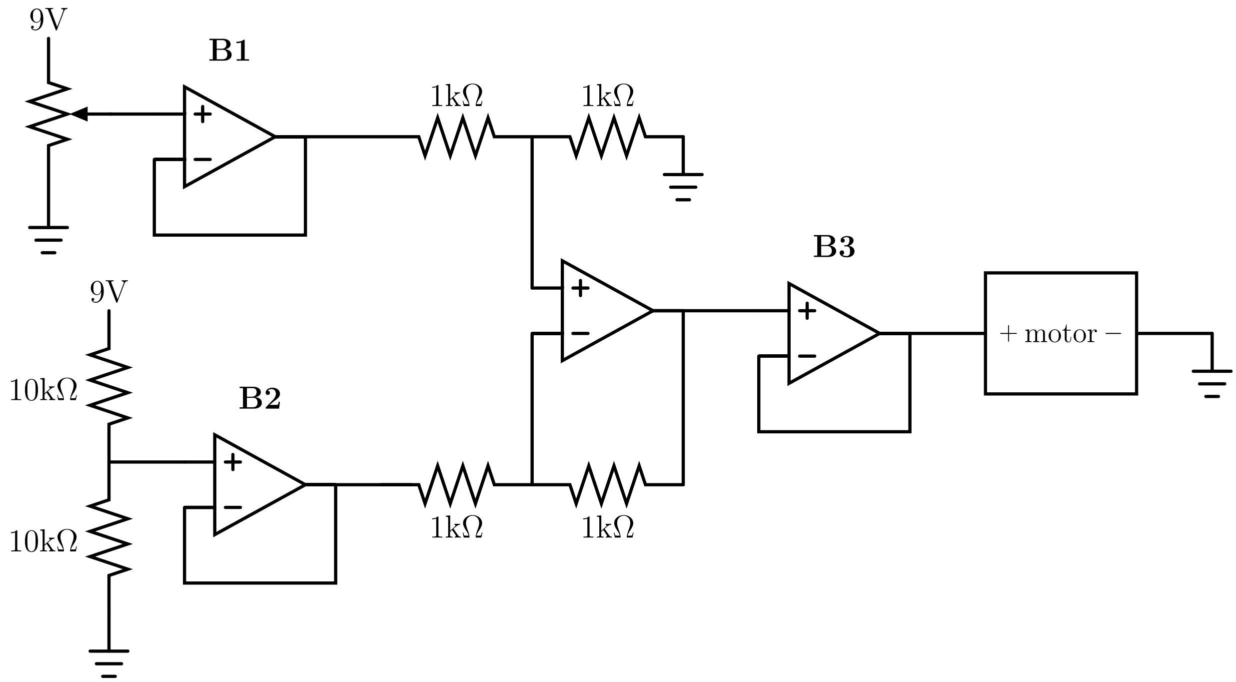
- Determine the maximum (most positive) and minimum (most negative) voltage that can be applied across the motor by adjusting α on the potentiometer.
- Each circuit contains some number of buffers, each labeled with B_i, $i = 1, 2 \cdots$. For each circuit, indicate which of the buffers (if any) are unnecessary in the sense that if they were removed the voltage applied to the motor would not change. If all buffers are necessary, write "None" in the box.

(2A) Circuit #1 is shown below.



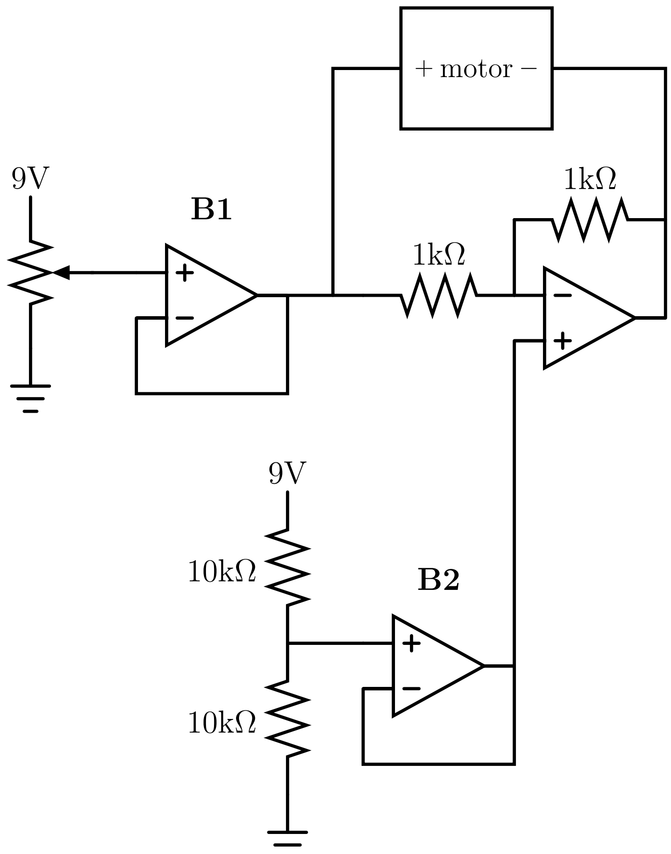
Maximum Motor Voltage:
Minimum Motor Voltage:
Unnecessary Buffers:

(2B) Circuit #2 is shown below.



Maximum Motor Voltage:
Minimum Motor Voltage:
Unnecessary Buffers:

(2C) Circuit #3 is shown below.

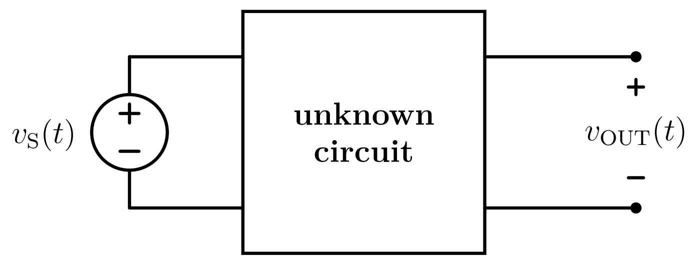


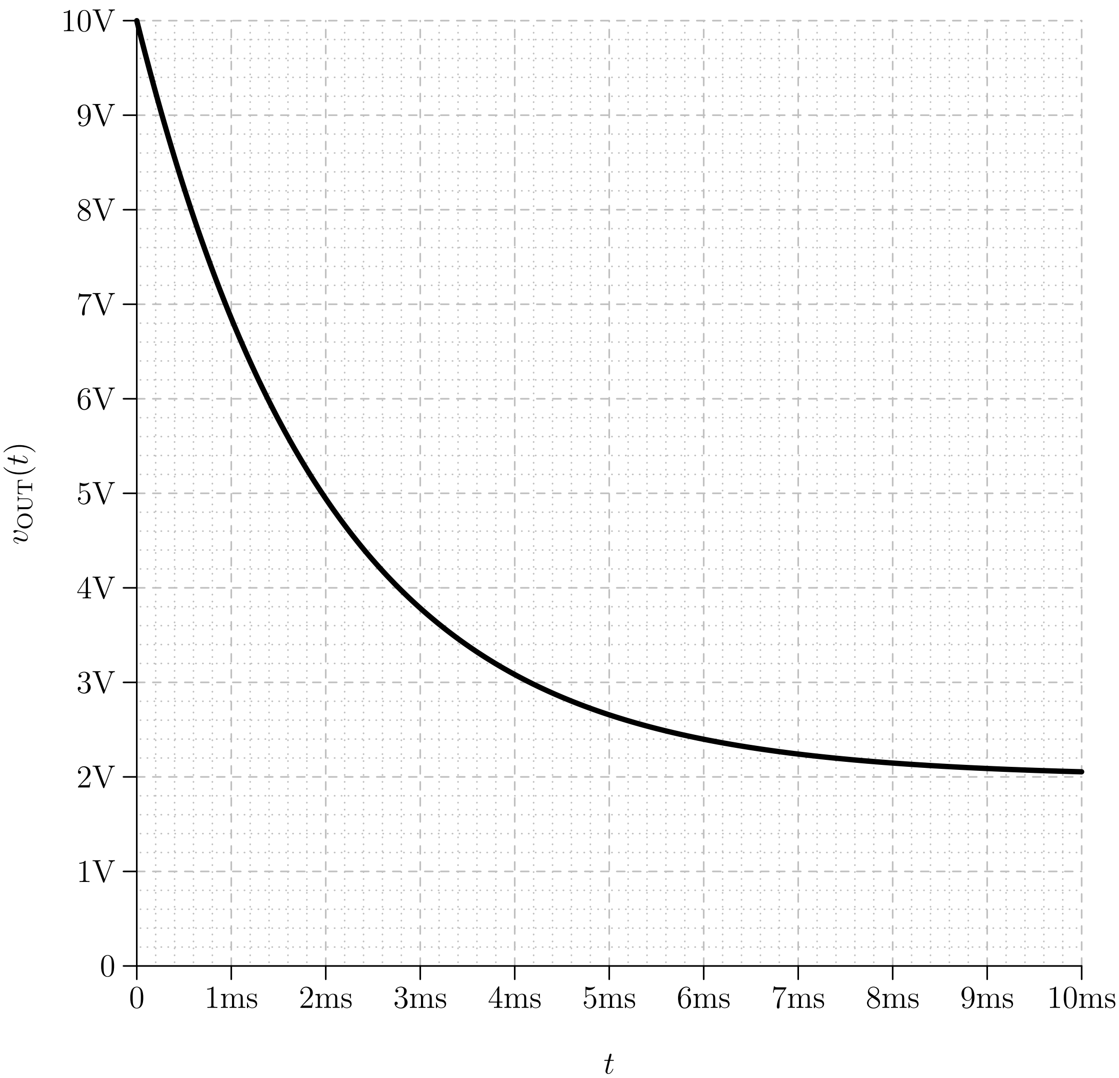
Maximum Motor Voltage:	
Minimum Motor Voltage:	
Unnecessary Buffers:	

Problem 3: Circuit Matching – 20%

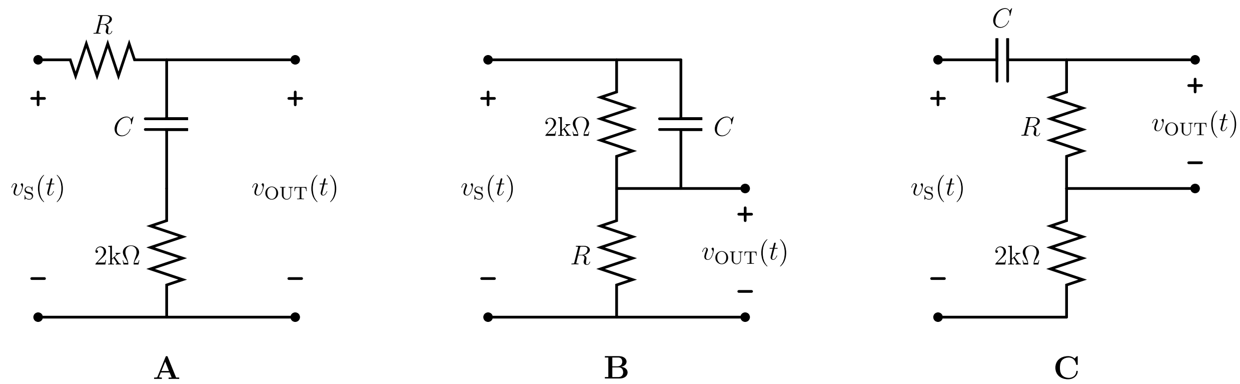
Consider the system shown below comprising a voltage source $v_S(t)$ and an unknown circuit. Prior to $t = 0$ the system has been at rest for a very long time with $v_S(t) = 0$. At $t = 0$, $v_S(t)$ takes a step as indicated below. The step excitation gives rise to the output voltage $v_{OUT}(t)$ also shown below.

$$v_S(t) = \begin{cases} 0\text{V} & \text{if } t < 0 \\ 10\text{V} & \text{otherwise} \end{cases}$$





(3A) Three possible forms (A,B,C) of the unknown circuit are shown below. By circling their letter designator (A,B,C) select all those that could yield the output shown above. Additionally, clearly explain your reasoning in the answer box provided below.

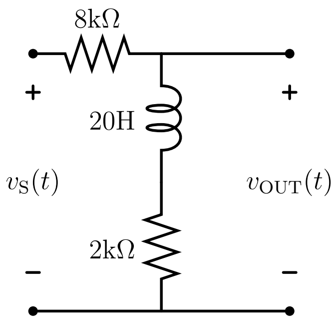


Reasoning:

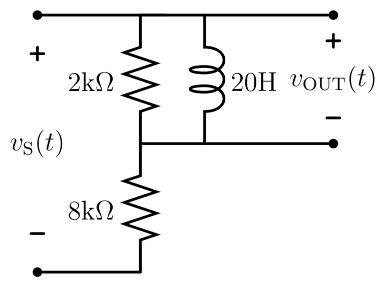
(3B) For each circuit selected above, determine the unknown resistance R and capacitance C. Enter the values in the corresponding answer box below, leaving the box empty for any non-selected circuits. *Proper units are expected.*

Circuit A:
Circuit B:
Circuit C:

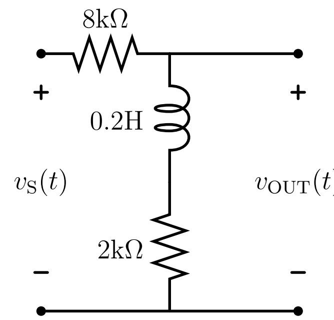
(3C) Three new possible forms (D,E,F) of the unknown circuit are shown below. By circling their letter designator (D,E,F) select all those that could yield the output shown above. Again, clearly explain your reasoning in the answer box provided below.



D



E

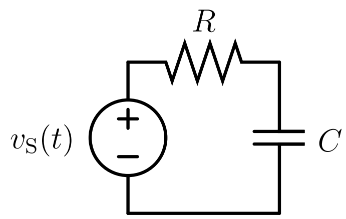


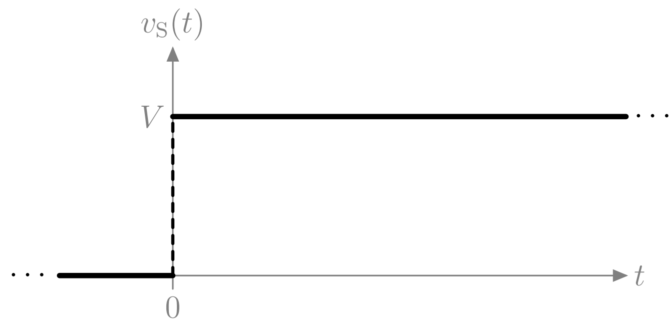
F

Reasoning:

Problem 4: Energy – 18%

Shown below is a source-resistor-capacitor network in which the source voltage $v_S(t)$ takes a step from 0V to V at $t = 0$. Prior to the step the source voltage was zero for a very long time.





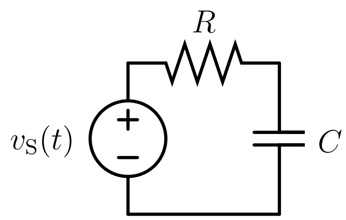
(4A) Determine the charge Q delivered by the source to the capacitor by the end of the transient, that is, at $t = \infty$.

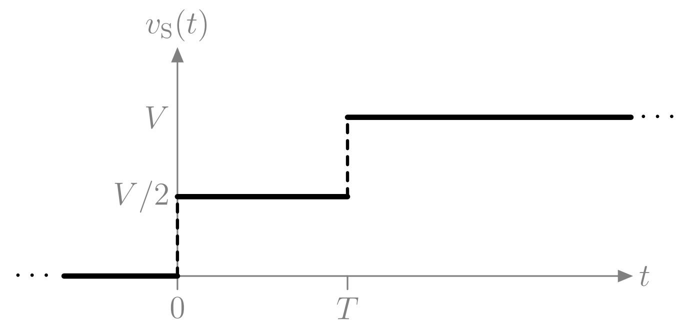
$Q =$

(4B) Determine the energy W_S delivered by the source, the energy W_R dissipated in the resistor, and the energy W_C delivered to the capacitor by the end of the transient. In addition, determine the energy delivery efficiency $\eta \equiv W_\mathrm{C}/W_\mathrm{S}$.

$W_\mathrm{S} =$
$W_\mathrm{R} =$
$W_\mathrm{C} =$
$\eta =$

(4C) Now consider the case in which the source voltage takes two half steps in rising from 0 V to V, as shown below. Assume that the time step T is very long such that $T \gg RC$ and the first step transient fully settles before the second step transient begins. Again determine the energy W_S delivered by the source, the energy W_R dissipated in the resistor, the energy W_C delivered to the capacitor, and the energy delivery efficiency η evaluated at the end of the two-step transient.





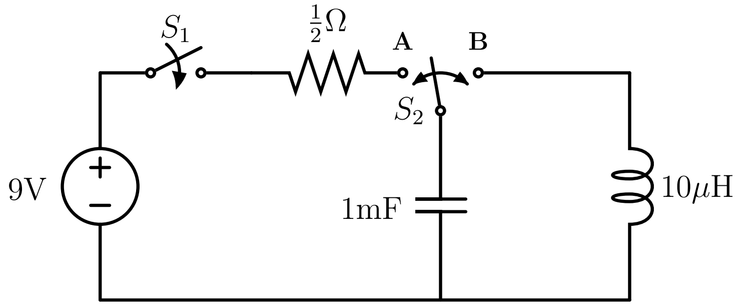
$W_S =$
$W_R =$
$W_C =$
$\eta =$

(4D) Explain why the two energy delivery efficiencies found above are not the same, and the reason for one being greater than the other.

Explanation:

Problem 5: LC Switching – 22%

Consider the circuit shown below.



The switches in the circuit act according to the following switching pattern.

- For all time $t < 1$ ms, switch S_1 is **open** and switch S_2 is at position **A**. Before $t = 1$ ms, the capacitor voltage and the inductor current are both zero.
- At $t = 1$ ms, switch S_1 **closes** and remains closed forever.
- At $t = 2$ ms, switch S_2 moves from position A to position **B**.
- At $t = 3$ ms, switch S_2 moves from position B back to position **A** and remains there forever.

On the axes below, sketch v_c and i_c as functions of time. Label the approximate value (with units) of v_c and i_c at each of the labeled times. Feel free to make reasonable approximations like $e = \pi = 3$.